Credit Index Calibration: How Do Models Perform ?

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Credit Index Calibration: How Do Models Perform ?
Zeliade Systems Technical Documentation, CDO series n. 1

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Related Material

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Zeliade Credit Analytics Library: Tranche Pricing Algorithm, CDO Series, n. 2.

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Summary

The correlation market evolves very fast. Most practitioners are in the process of updating their methodologies.

The Random Factor Loadings (RFL) strategy has ushered in a new era in multiname Credit Risk analysis and valuation.

Yet RFL models fail to grasp some essential features of the correlation market. Their fit to market data is still not satisfactory.

Zeliade's proprietary model for STCDO, an enhanced RFL model, captures the whole correlation structure.

I. INTRODUCTION

This paper contains a comparative analysis of calibrations performed on Credit Index tranches (ItraxxS35Y, 22 Mar. 2005 to 22 Apr. 2005) with three models:

- the one-factor Gaussian copula (OGC) model,
- the Andersen/Sidenius or Random Factor Loadings (RFL) model and,
- an enhanced Random Factor Loadings model. In the remainder of this document, this enhanced RFL model is referred to as the Zeliade model.

II. THE OGC MODEL

The one factor Gaussian copula model is often presented as the multiname credit risk analogue of the Black-Scholes model for equity derivatives.

Its failure to account for the real structure of CDOs is illustrated by the correlation skews of the compound and base correlation curves (see *Figure 1*and *Figure 2*), that would need to be flat for the model to reflect the real behaviour of multiname credit portfolios.

Figure 1 – Compound correlation curve, ItraxxS35Y as of 08 Apr. 2005

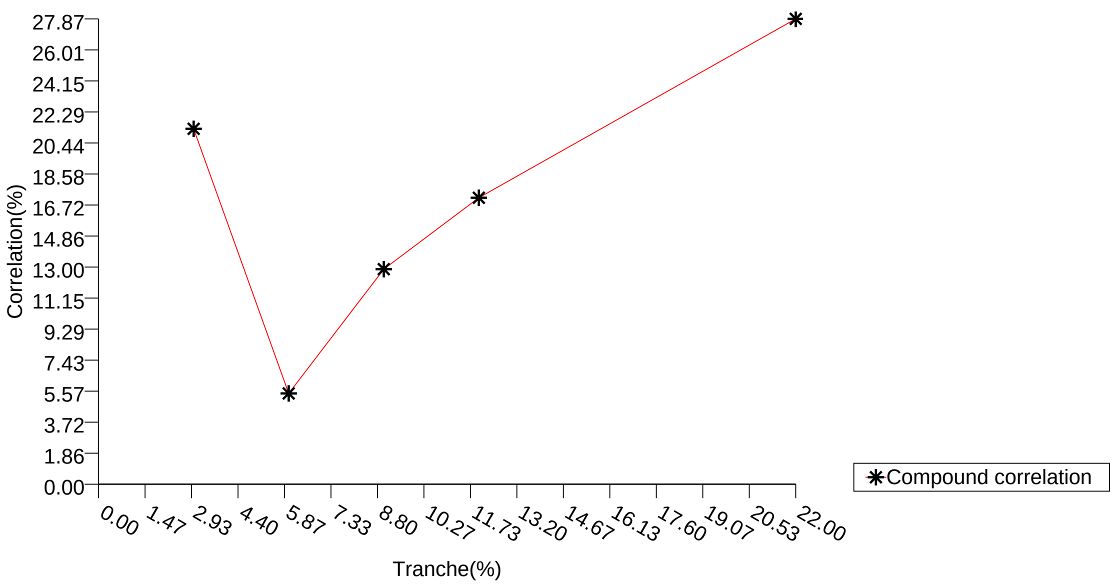
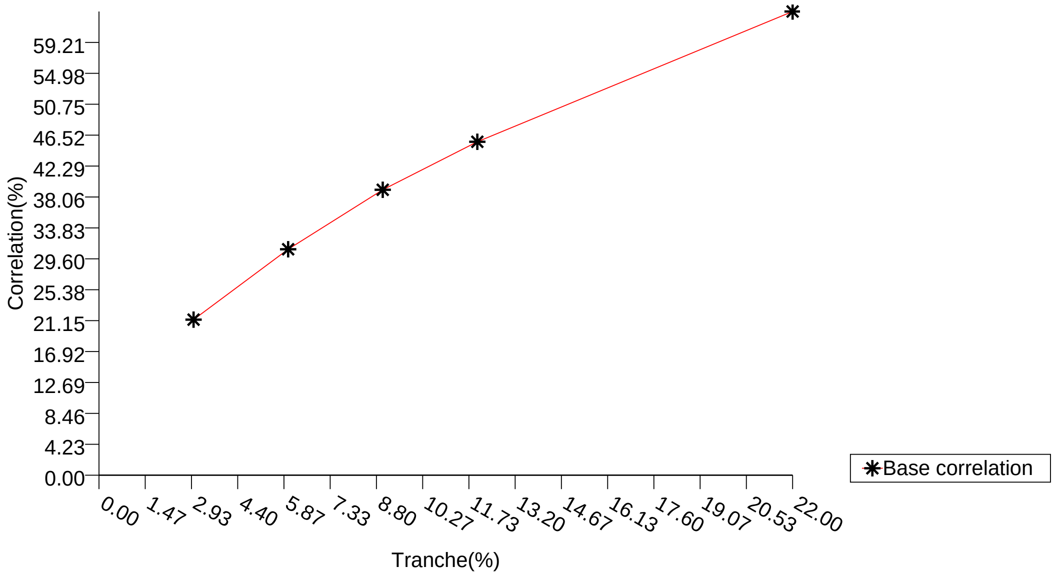


Figure 2 – Base correlation curve, ItraxxS35Y as of 08 Apr. 2005



The OGC model is deficient when it comes to manage simultaneously the various tranches.

This is easily demonstrated by calibrating the model to the whole structure of a CDO (see *Figure 3* and *Figure 4*).

Figure 3 – OGC: Spread calibration error history (ItraxxS35Y)

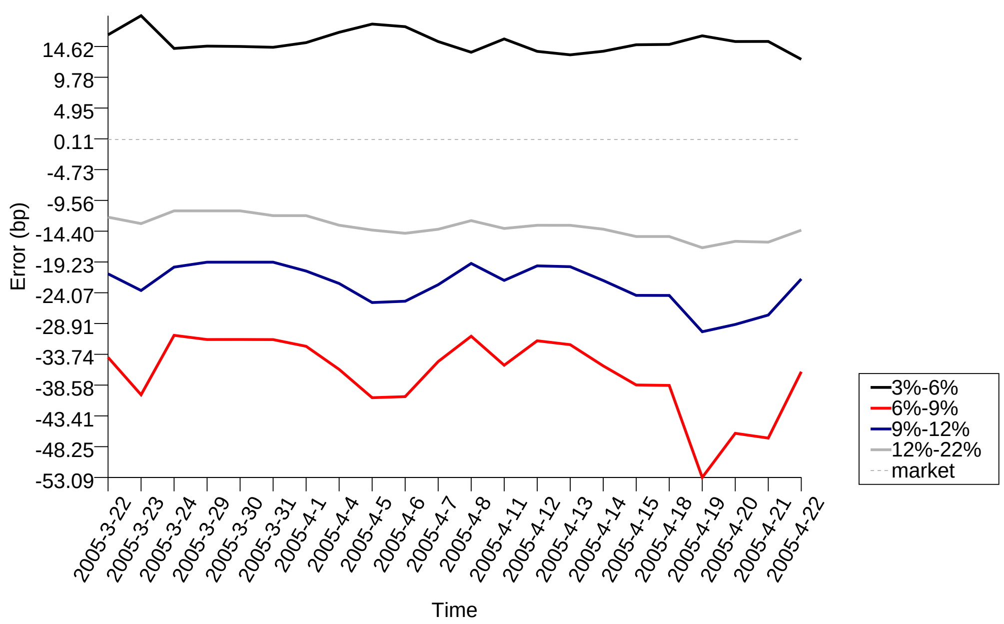
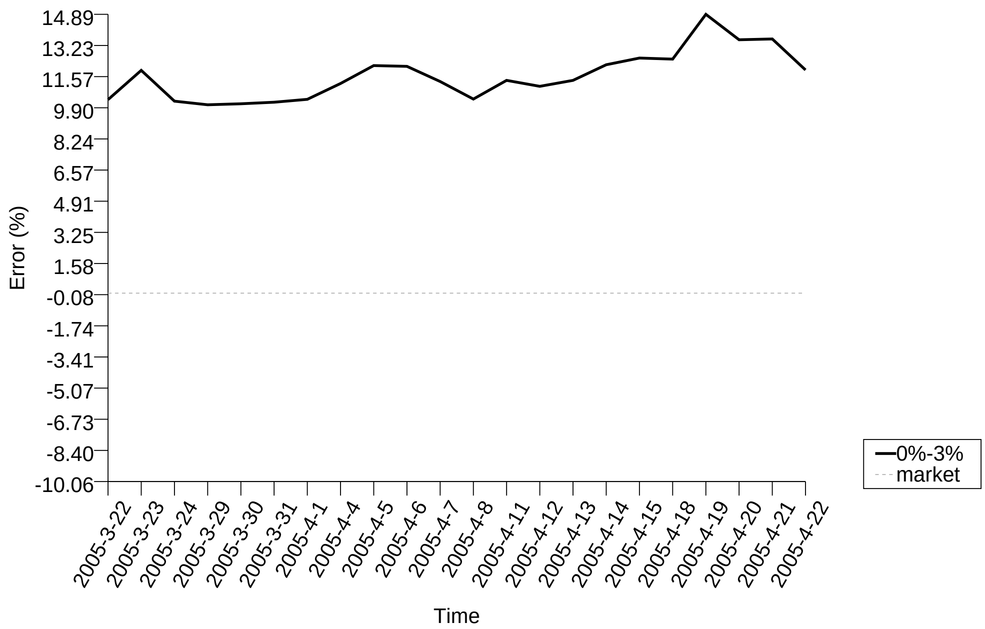


Figure 4 – OGC: Upfront calibration error history (ItraxxS35Y)



These results should be compared with the bid/ask levels. For example:

Table 1 – Bid/Ask levels

	Upfront equity	3-6%	6-9%	9-12%	12-22%
08 Apr. 2005	22.8 / 23.2 %	147.25 / 149.25 bp	45 / 47 bp	19.5 / 22 bp	11.275 / 14.275 bp
19 Apr. 2005	30.15 / 30.7 %	189 / 192 bp	61 / 66 bp	28 / 33 bp	15.5 / 18.5 bp

III. THE RFL MODEL

The Random Factor Loading (RFL) strategy has been introduced by Bank of America's Andersen and Sidenius in June 2004 in order to account for the correlation skews.
The concept of regime correlation has been successful and has ushered in a new era in multiname Credit Risk analysis and valuation.

As illustrated by the graphs displayed in *Figure 5* and *Figure 6*, the fit of the RFL model to market data is certainly much better than the one using OGC, justifying the attention it has received from quantitative research groups since its introduction.

Figure 5 – RFL: Spread calibration error history (ItraxxS35Y)

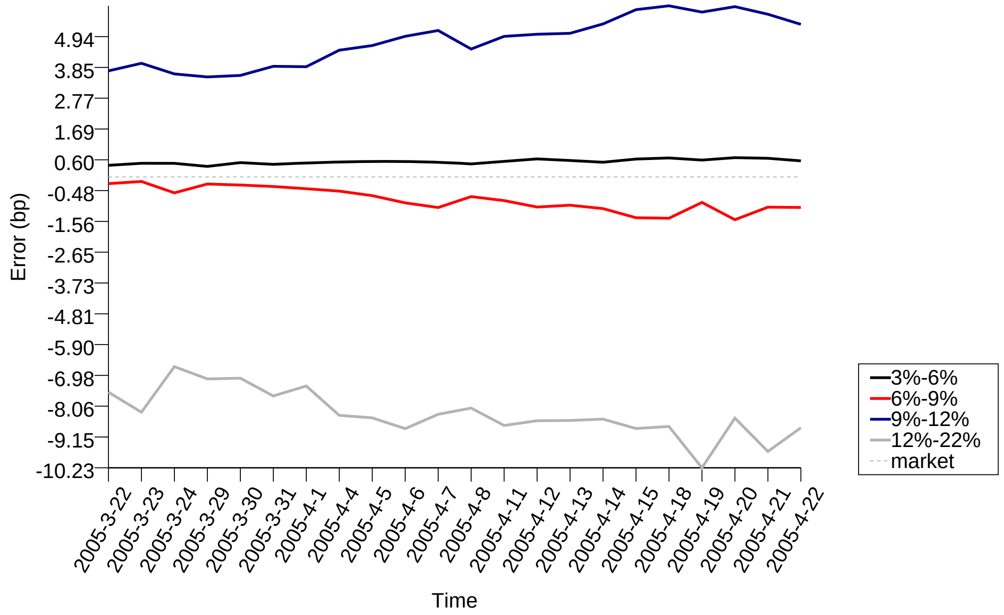
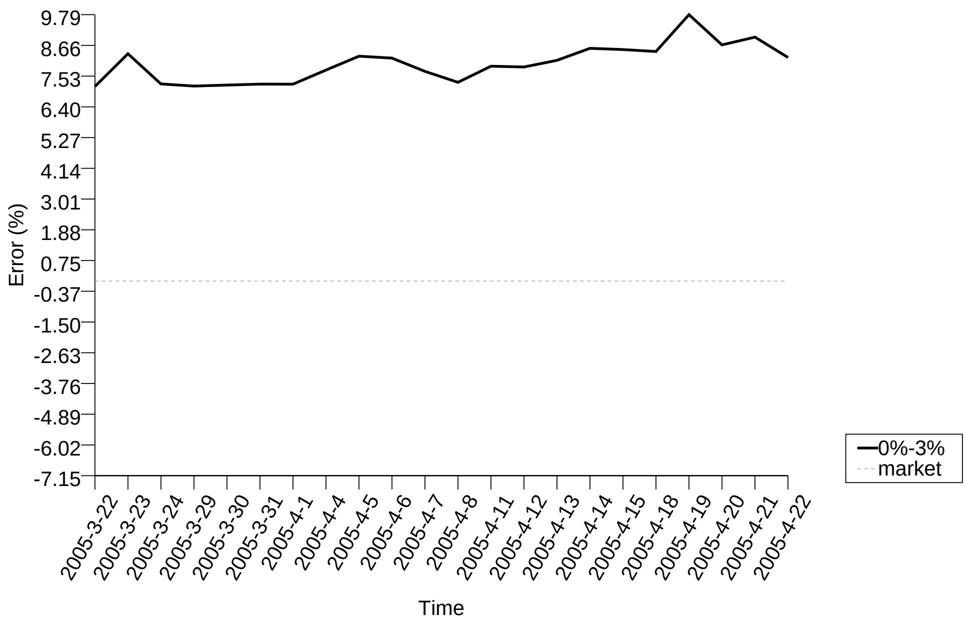


Figure 6 – RFL: Upfront calibration error history (ItraxxS35Y)



However, the fit is still not good enough to provide STCDO managers with the tools they would expect. Except on the 3-6% junior-mezzanine and 6-9% mezzanine tranches, the fit is poor:

- *Mezzanine, 9-12% tranche:* the error is about two times the bid/ask spread,

- *Senior, 12-22% tranche:* the error is close to the price of the tranche,
- *Equity tranche:* the error amounts to 10 times the bid/ask spread, or one fourth of the (absolute) spread of the tranche.

IV. THE ZELIADE MODEL

Although relying on a recent analysis of macroeconomic phenomena related to the long term behaviour of market fundamentals, the RFL model fails to grasp some other essential features of the correlation market. These are taken into account in Zeliade's proprietary model.

The Zeliade model is an enhancement of the RFL model that yields a much better fit to market data, as shown in *Figure 7* and *Figure 8*. Our model is therefore able to provide, for example, an accurate modelling of hedging parameters, as well as a sound basis for the pricing of complex products such as CDO^2s.

Figure 7 – Zeliade model: Spread calibration error history (ItraxxS35Y)

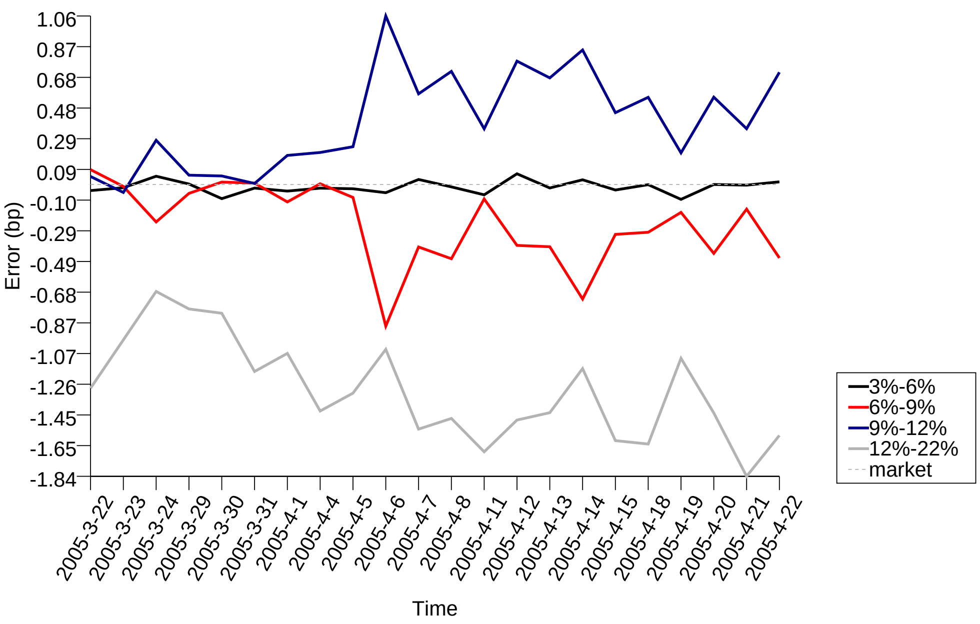
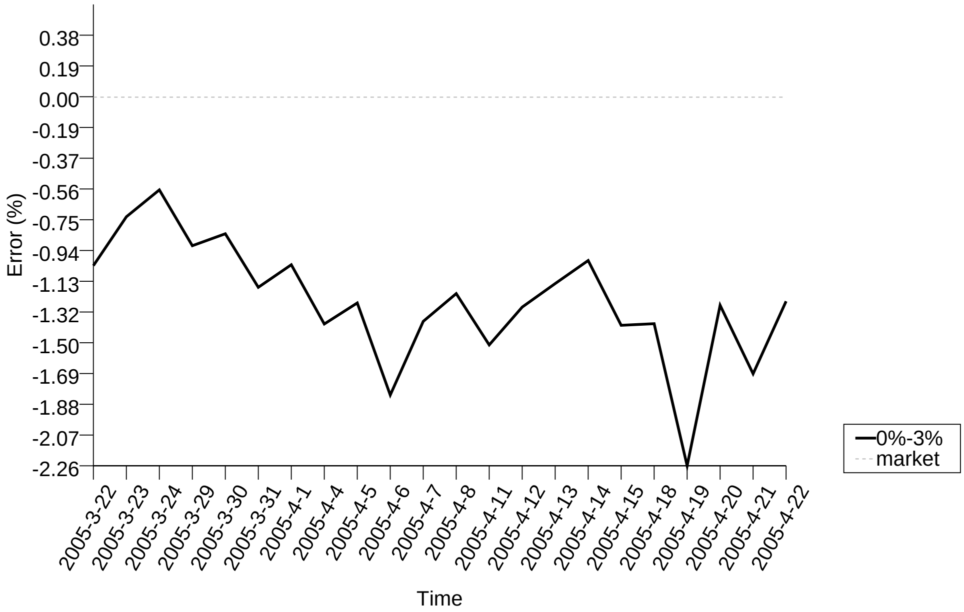


Figure 8 – Zeliade model: Upfront calibration error history (ItraxxS35Y)



As for the OGC and RFL models, the calibration of Zeliade's model behaves steadily and consistently over the period.

Furthermore, the calibration results are much better than those of the RFL model:

- *Junior mezzanine, 3-6% tranche:* the fit is almost perfect,
- *Other mezzanine tranches:* the calibration error is less than 1 bp on the 6-9% and 9-12% tranches (less than the bid/ask spreads). This should be compared to the 4 to 5 bp error of the RFL model on the 9-12% tranche, as shown in *Figure 5*,
- *Senior, 12-22% tranche:* usually the most difficult tranche to fit. The error is less than 2 bp, once again less than the bid/ask spread. This result is particularly impressive when compared to the RFL model, with an error of 6 to 10 bp, that is around 50% of the value of the tranche,
- *Equity tranche*: the error remains below 2%, most of the time around 1%, a size of the same order as the one of the (tiny) bid/ask spread. Once again, this result contrasts with the 7 to 10% error of the RFL model.

V. METHODOLOGY

The very efficient computational tools that have been developped by Zeliade allow to avoid using the Large Pool model that only delivers a crude approximation, particularly when it comes to the pricing of bespoke tranches.

All the computations in the present document have been done with the real (market, idiosyncratic, heterogeneous) CDS spreads and with homogeneous recovery rates of 40%, a standard assumption.

In all cases, the calibration procedure minimizes a least-square error between the model and market prices of tranches.

VI. SPEED

There is no tradeoff between the calibration accuracy performance of the Zeliade model and computation time.

Zeliade's core pricing algorithm relies on an enhanced saddle-point method and computes several tranches per second on a standard laptop. This algorithm allows computing a joint index tranches calibration in typically 30 seconds with no special hardware or distributed computing technology.
This is *hundred times* faster than Monte Carlo pricing.

VII. COMPARATIVE BEHAVIOUR OF THE THREE MODELS

The next graphs summarize the calibration results for the three models on the various tranches.

Figure 9 – Joint calibration error history (ItraxxS35Y): equity upfront, 0-3%

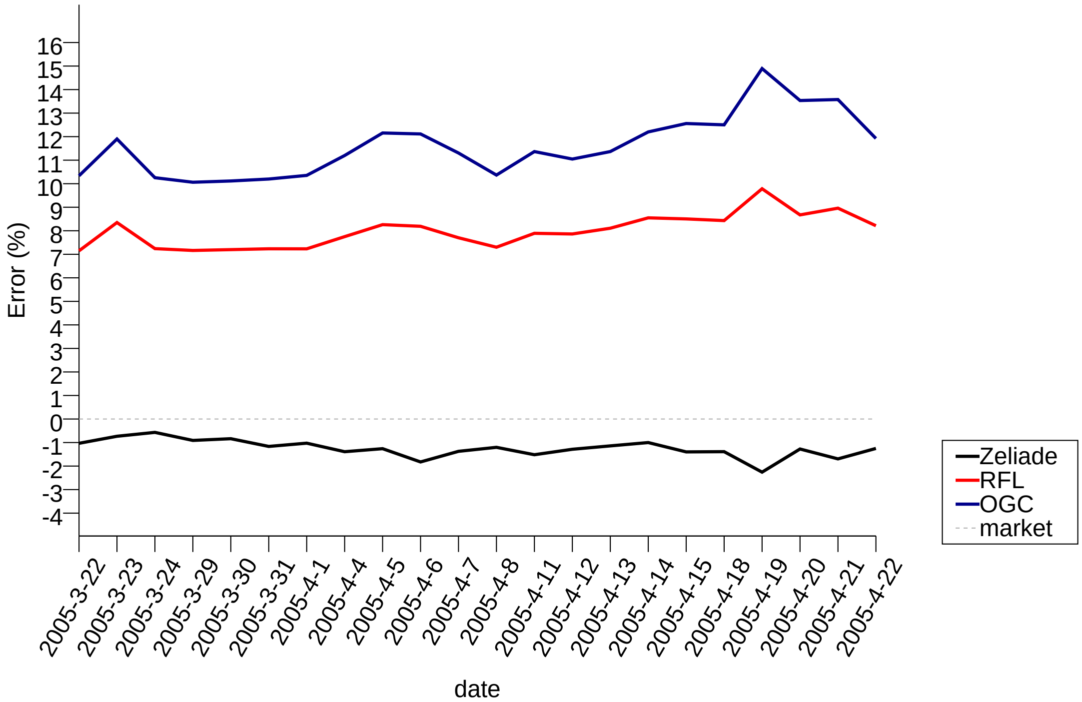


Figure 10 – Joint calibration error history (ItraxxS35Y): junior mezzanine spread, 3-6%

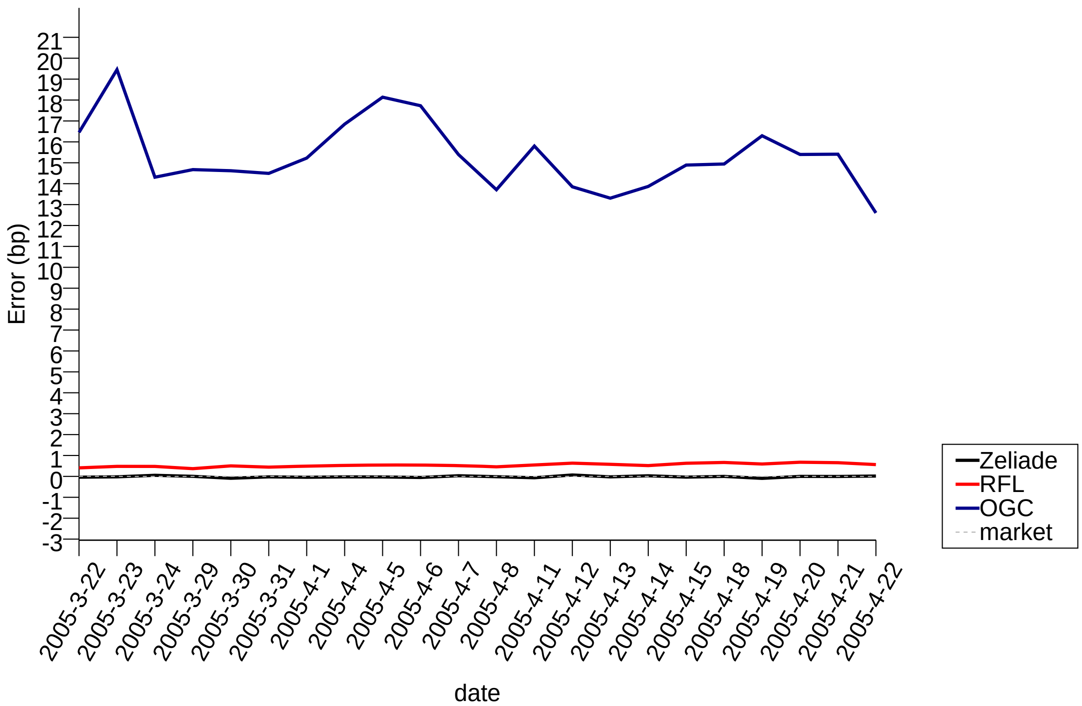


Figure 11 – Joint calibration error history (ItraxxS35Y): mezzanine spread, 6-9%

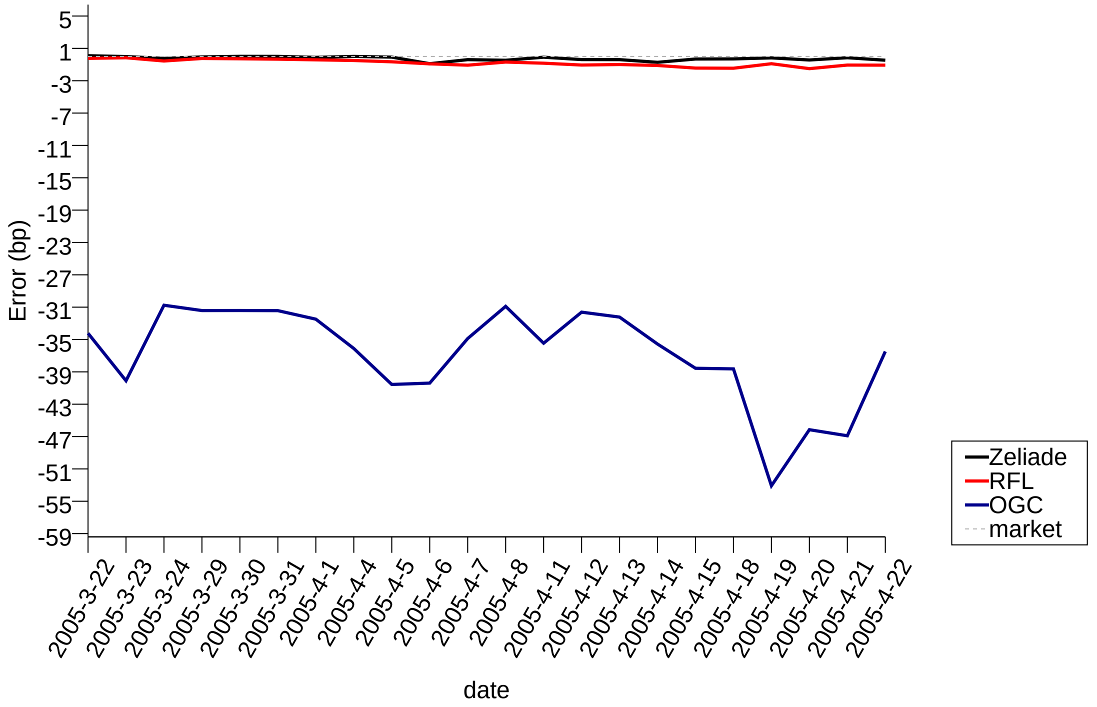


Figure 12 – Joint calibration error history (ItraxxS35Y): mezzanine spread, 9-12%

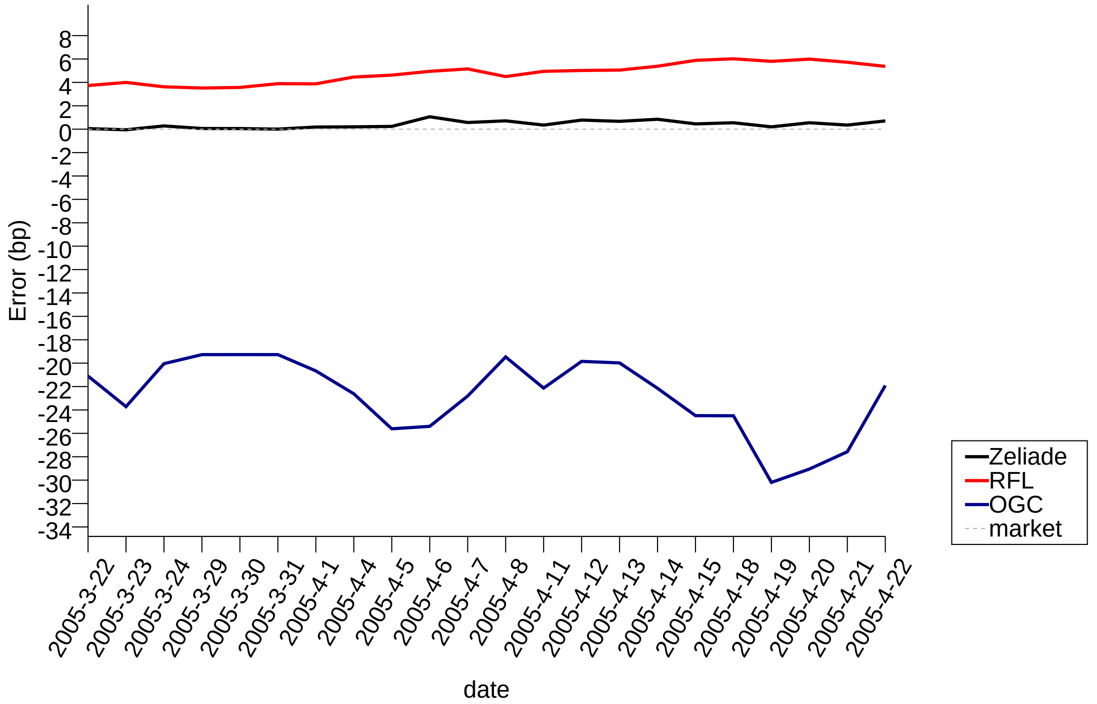
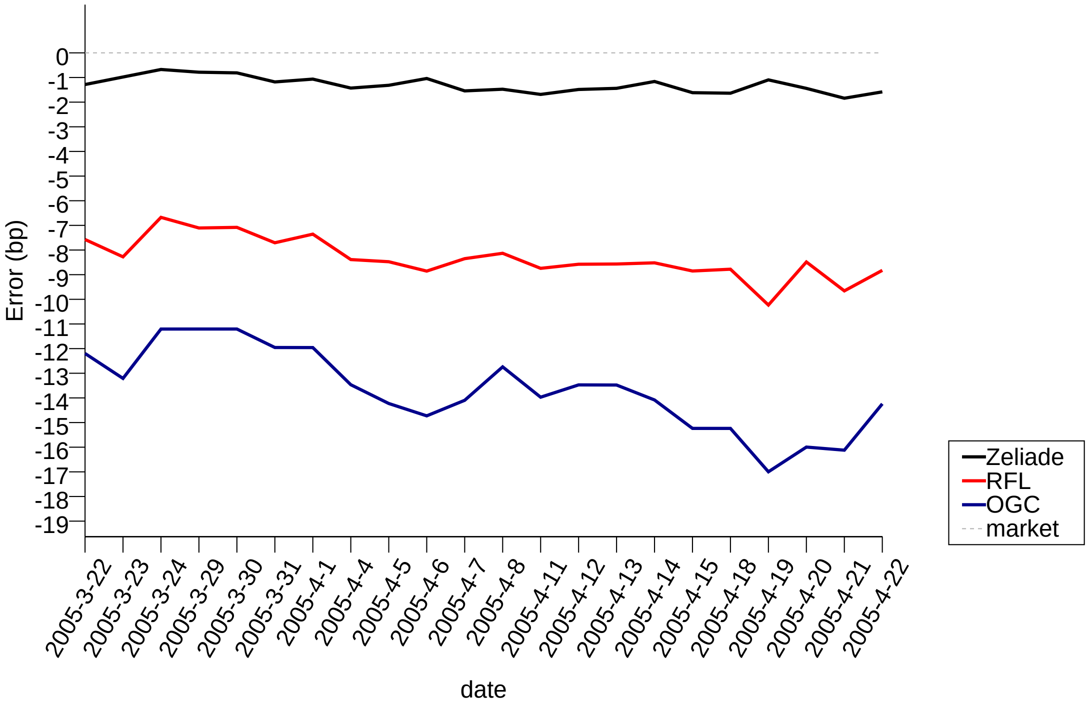


Figure 13 – Joint calibration error history (ItraxxS35Y): senior spread, 12-22%



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