



Zeliade Credit Analytics Library: Tranche Pricing Algorithm

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RELATED MATERIAL

Please contact Zeliade Systems to obtain the following related document:
Credit Index Calibration: How Do Models Perform ?, CDO Series, n. 1.

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Summary

A fast and reliable method for the computation of the (tranching) loss expectancy is needed to price bespoke deals and calibrate tranching indexes.

Monte Carlo algorithms are reliable, but not fast enough. The Quantization probabilistic method provides reference prices but is still not fast enough.

The fastest known way to compute tranche prices in the case of heterogeneous underlying portfolios is the Saddle-Point method. In its standard flavor, accuracy is not satisfactory.

Zeliade Tranche Pricing Algorithm enhances the Saddle-Point method at the algorithmic and numerical levels, yielding improved speed and accuracy.

I. INTRODUCTION

In order to price bespoke deals and calibrate tranching indexes, one needs a fast and reliable method for the computation of the (tranching) loss expectancy:

- Monte Carlo algorithms are reliable, but not fast enough.
- Quantization, a more elaborate probabilistic method that approximates a loss law at a given time with a prescribed accuracy is even more reliable but still not fast enough.
However prices obtained by Quantization method can be safely regarded as reference prices.
- The fastest known way to compute tranche prices in the case of heterogeneous underlying portfolios is the Saddle-Point method.
The accuracy of the classical Saddle-Point method is not satisfactory.
- The Zeliade Tranche Pricing Algorithm enhances the standard Saddle-Point method both at the algorithmic and numerical level yielding improved speed and accuracy.

Computation experiments timed in this document were performed on a mainstream laptop (AMD Athlon™ XP 2600 with 1.14Ghz processor and 512Mo RAM).

II. SPEED

In terms of speed, there is a sharp difference between the Quantization method and the Zeliade variant of the Saddle-Point method:

Table 1 – Speed results

| Tranche | | Running Time | |
|---------|-------|-------------------------------|--------------|
| Lower | Upper | Zeliade enhanced Saddle Point | Quantization |
| 0 % | 3 % | 0.14 sec. | 9.03 sec. |
| 3 % | 6 % | 0.14 sec. | 9.06 sec. |
| 6 % | 9 % | 0.14 sec. | 9.03 sec. |
| 9 % | 12 % | 0.12 sec. | 9.03 sec. |
| 12 % | 22 % | 0.12 sec. | 9.06 sec. |

III. CALIBRATION

The difference we have seen above implies significant speed improvement for the calibration process:

Table 2 – Speed of the calibration process

| | Zeliade enhanced Saddle Point | Quantization |
|-----------|-------------------------------|--------------|
| DJ Itraxx | 81.9 sec. | 1314.1 sec. |
| DJ CDX | 46.4 sec. | 634.6 sec. |

IV. ACCURACY

We present below comparative pricing outputs of both Saddle-Point and Quantization methods:

Table 3 – Accuracy (parameters calibrated on the DJ Itraxx Europe 5 Year, 20Jun-05)

| Tranche | | Model Mid | | Difference |
|---------|-------|-------------------------------|--------------|------------|
| Lower | Upper | Zeliade enhanced Saddle Point | Quantization | |
| 0 % | 3 % | 17.073 % | 17.207 % | 0.134 |
| 3 % | 6 % | 112.561 bp | 112.558 bp | - 0.003 |
| 6 % | 9 % | 36.033 bp | 35.981 bp | - 0.052 |
| 9 % | 12 % | 18.499 bp | 18.695 bp | 0.196 |
| 12 % | 22 % | 8.529 bp | 8.458 bp | - 0.071 |

Table 4 – Accuracy (parameters calibrated on the DJ CDX.NA.IG 5 Year, 20-Jun-05)

| Tranche | | Model Mid | | Difference |
|---------|-------|-------------------------------|--------------|------------|
| Lower | Upper | Zeliade enhanced Saddle Point | Quantization | |
| 0 % | 3 % | 30.545 % | 30.563 % | 0.018 |
| 3 % | 7 % | 159.620 bp | 159.438 bp | - 0.182 |
| 7 % | 10 % | 52.300 bp | 52.465 bp | 0.165 |
| 10 % | 15 % | 18.962 bp | 18.687 bp | - 0.275 |
| 15 % | 30 % | 8.196 bp | 8.170 bp | - 0.026 |

The good fit between the mids obtained through the two different techniques becomes apparent.

It is worth mentioning that Zeliade improvements to the SaddlePoint algorithm make it more accurate even in the case of Mezzanine tranches.



V. CONCLUSION

The Zeliade Tranche Pricing Algorithm, which relies on an enhanced Saddle-Point method, is both very accurate and exceptionally fast. This allows a full tranching index calibration within a minute on a standard laptop.

It also dramatically reduces the time needed for portfolio selection algorithms, hedge and VaR simulation or historical backtests which thus become feasible.



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