Credit Index Calibration: How Do Models Perform?

November 2005

zelia.de systems
Credit Index Calibration: How Do Models Perform?
Zeliade Systems Technical Documentation, CDO series n. 1
© 2005-2006 Zeliade Systems

TRADEMARKS
Zeliade Systems is a registered trademark of Zeliade Systems S.A.S.
All other products or services mentioned in this document are covered by the trademarks, service marks, or product names as designated by their owners.

RELATED MATERIAL
Please contact Zeliade Systems to obtain the following related document:
Zeliade Credit Analytics Library: Tranche Pricing Algorithm, CDO Series, n. 2.

DISCLAIMER
No part of this publication may be reproduced, stored in a retrieval system or transmitted, in any form or by any means, photocopying, recording or otherwise, without written consent of Zeliade Systems.
While every precaution has been taken in the preparation of this material, Zeliade Systems assumes no responsibility for errors or omissions. The publication and features described herein are subject to change without notice due to continuous progress in methodology, design and manufacturing.
No third party patent liability is assumed with respect to the use of the information contained herein.
Zeliade Systems shall have no liability for any errors or damage of any kind in connection with the furnishing or use of this material.
I. **INTRODUCTION**

This paper contains a comparative analysis of calibrations performed on Credit Index tranches (ItraxxS35Y, 22 Mar. 2005 to 22 Apr. 2005) with three models:

- the one-factor Gaussian copula (OGC) model,
- the Andersen/Sidenius or Random Factor Loadings (RFL) model and,
- an enhanced Random Factor Loadings model. In the remainder of this document, this enhanced RFL model is referred to as the Zeliade model.

II. **THE OGC MODEL**

The one factor Gaussian copula model is often presented as the multination credit risk analogue of the Black-Scholes model for equity derivatives.

Its failure to account for the real structure of CDOs is illustrated by the correlation skews of the compound and base correlation curves (see Figure 1 and Figure 2), that would need to be flat for the model to reflect the real behaviour of multination credit portfolios.
The OGC model is deficient when it comes to manage simultaneously the various tranches.

This is easily demonstrated by calibrating the model to the whole structure of a CDO (see Figure 3 and Figure 4).
These results should be compared with the bid/ask levels. For example:

<table>
<thead>
<tr>
<th>Date</th>
<th>Upfront equity</th>
<th>3-6%</th>
<th>6-9%</th>
<th>9-12%</th>
<th>12-22%</th>
</tr>
</thead>
<tbody>
<tr>
<td>08 Apr. 2005</td>
<td>22.8 / 23.2 %</td>
<td>147.25 / 149.25 bp</td>
<td>45 / 47 bp</td>
<td>19.5 / 22 bp</td>
<td>11.275 / 14.275 bp</td>
</tr>
<tr>
<td>19 Apr. 2005</td>
<td>30.15 / 30.7 %</td>
<td>189 / 192 bp</td>
<td>61 / 66 bp</td>
<td>28 / 33 bp</td>
<td>15.5 / 18.5 bp</td>
</tr>
</tbody>
</table>

### III. THE RFL MODEL

The Random Factor Loading (RFL) strategy has been introduced by Bank of America’s Andersen and Sidenius in June 2004 in order to account for the correlation skews. The concept of regime correlation has been successful and has ushered in a new era in multiname Credit Risk analysis and valuation.

As illustrated by the graphs displayed in Figure 5 and Figure 6, the fit of the RFL model to market data is certainly much better than the one using OGC, justifying the attention it has received from quantitative research groups since its introduction.
However, the fit is still not good enough to provide STCDO managers with the tools they would expect. Except on the 3-6% junior-mezzanine and 6-9% mezzanine tranches, the fit is poor:

- **Mezzanine, 9-12% tranche:** the error is about two times the bid/ask spread,
• Senior, 12-22% tranche: the error is close to the price of the tranche,
• Equity tranche: the error amounts to 10 times the bid/ask spread, or one fourth of the (absolute) spread of the tranche.

IV. THE ZELIADE MODEL

Although relying on a recent analysis of macroeconomic phenomena related to the long term behaviour of market fundamentals, the RFL model fails to grasp some other essential features of the correlation market. These are taken into account in Zeliade’s proprietary model.

The Zeliade model is an enhancement of the RFL model that yields a much better fit to market data, as shown in Figure 7 and Figure 8. Our model is therefore able to provide, for example, an accurate modelling of hedging parameters, as well as a sound basis for the pricing of complex products such as CDO2’s.

Figure 7 – Zeliade model: Spread calibration error history (traxxS35Y)
As for the OGC and RFL models, the calibration of Zeliade’s model behaves steadily and consistently over the period.

Furthermore, the calibration results are much better than those of the RFL model:

- **Junior mezzanine, 3-6% tranche**: the fit is almost perfect,
- **Other mezzanine tranches**: the calibration error is less than 1 bp on the 6-9% and 9-12% tranches (less than the bid/ask spreads). This should be compared to the 4 to 5 bp error of the RFL model on the 9-12% tranche, as shown in Figure 5,
- **Senior, 12-22% tranche**: usually the most difficult tranche to fit. The error is less than 2 bp, once again less than the bid/ask spread. This result is particularly impressive when compared to the RFL model, with an error of 6 to 10 bp, that is around 50% of the value of the tranche,
- **Equity tranche**: the error remains below 2%, most of the time around 1%, a size of the same order as the one of the (tiny) bid/ask spread. Once again, this result contrasts with the 7 to 10% error of the RFL model.
V. METHODOLOGY

The very efficient computational tools that have been developed by Zeliade allow to avoid using the Large Pool model that only delivers a crude approximation, particularly when it comes to the pricing of bespoke tranches.

All the computations in the present document have been done with the real (market, idiosyncratic, heterogeneous) CDS spreads and with homogeneous recovery rates of 40%, a standard assumption.

In all cases, the calibration procedure minimizes a least-square error between the model and market prices of tranches.

VI. SPEED

There is no tradeoff between the calibration accuracy performance of the Zeliade model and computation time.

Zeliade’s core pricing algorithm relies on an enhanced saddle-point method and computes several tranches per second on a standard laptop. This algorithm allows computing a joint index tranches calibration in typically 30 seconds with no special hardware or distributed computing technology. This is hundred times faster than Monte Carlo pricing.

VII. COMPARATIVE BEHAVIOUR OF THE THREE MODELS

The next graphs summarize the calibration results for the three models on the various tranches.
Figure 9 – Joint calibration error history (iTraxxS35Y): equity upfront, 0-3%

Figure 10 – Joint calibration error history (iTraxxS35Y): junior mezzanine spread, 3-6%
Figure 11 – Joint calibration error history (iTraxxS35Y): mezzanine spread, 6-9%

Figure 12 – Joint calibration error history (iTraxxS35Y): mezzanine spread, 9-12%
Figure 13 – Joint calibration error history (ItraxxS35Y): senior spread, 12-22%
56 rue Jean-Jacques Rousseau – F-75001 Paris – France
Tel: +33 (0)1 40 26 17 29 – Fax: +33 (0)1 40 26 17 81
Email: contact@zeliade.com
Web: http://www.zeliade.com/